To prove: The equation $(p/q)^2 = 3$ has no solution for $p, q \in \mathbb{N}$.

Suppose that there are natural numbers $p$ and $q$ such that $(p/q)^2 = 3$. Without loss of generality, suppose further that $p$ and $q$ have no common divisors (for any such divisors could be cancelled as a preliminary step without affecting the truth of the equation).

Since $(p/q)^2 = 3$, $p^2 = 3q^2$. Hence 3 evenly divides $p^2$, and therefore also evenly divides $p$. Let $k$ be $p/3$; since 3 evenly divides $p$, $k$ is a natural number. So $9k^2 = (3k)^2 = p^2 = 3q^2$, and hence $3k^2 = q^2$. Therefore, 3 evenly divides $q^2$, and hence also evenly divides $q$. Thus $p$ and $q$ have 3 as a common divisor, contrary to our supposition.

The original supposition must therefore be false: There are no natural numbers $p$ and $q$ such that $(p/q)^2 = 3$. ■